Lecture 1

Text Book : Wiley International edition, 9th edition, Elementry linear Algebra
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Chapter 1

System of Linear Equations and Matrices

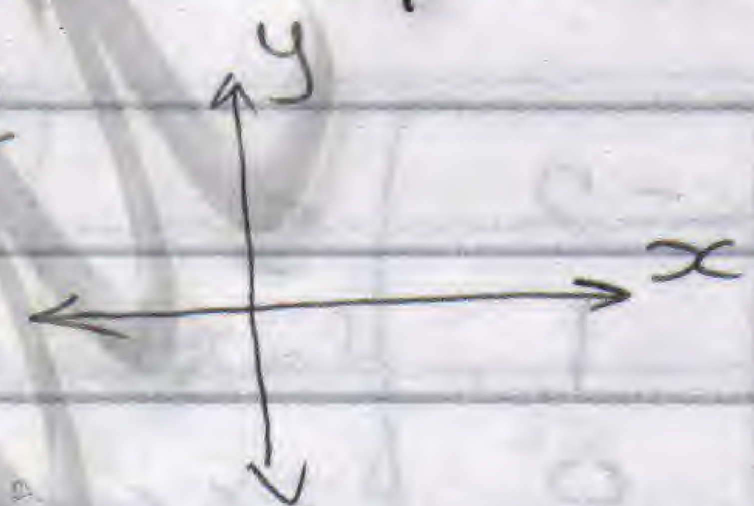
Eq. of st. Line $ax + by + c = 0$ ⟷ Linear equations

Ex.1 Linear or not :

$3x^2 + y + 7x = 0$	(NL)	$x + 4y + c = 0$	(L)
$x + xy + y = 7$	(NL)	$x + y = \sin\frac{\pi}{3}$	(L)
$x + y = \boxed{\sin x}$	(NL)	$x + \frac{1}{x} + y = 7$	(NL)

(1.1) Introduction to Systems of Linear Equations :

A System of Linear Equations



A st. Line in xy-Plane

$$a_1 x + b_1 y = b$$

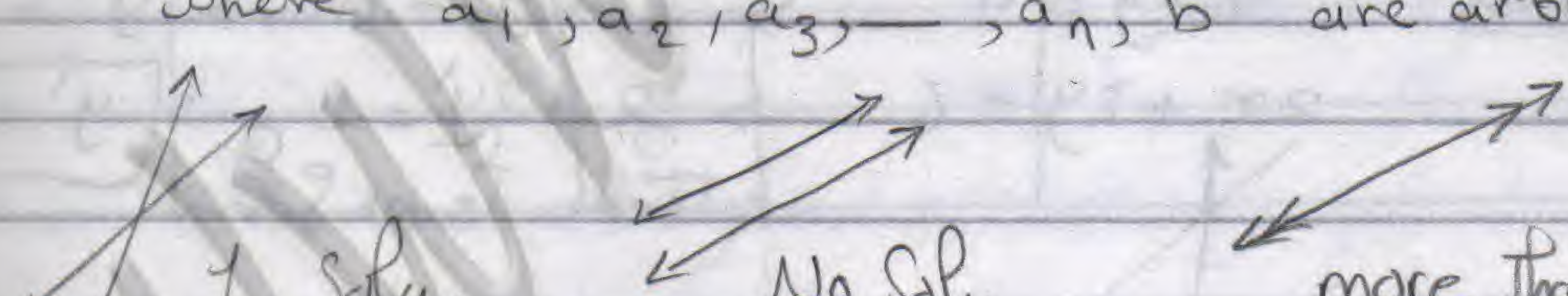
is a Linear equation where a_1, b_1, b are real Const.
Arbitary Const.

The linear Equation in the n Variables

$$x_1, x_2, \text{——}, x_n$$

$$a_1 x_1 + a_2 x_2 + a_3 x_3 + \text{——} + a_n x_n = b$$

where $a_1, a_2, a_3, \text{——}, a_n, b$ are arbitary Const.



1 Solu.   No Solu   more Than One Solu.

Ex.1 linear Or not

$x + 3y = 7$	(L)	$3x + 2y - z + xz = 4$	(NL)
$y = \frac{1}{2}x + 3z + 1$	(L)	$y = \sin x$	(NL)
$x_1 - 2x_2 - 3x_3 + x_4 = 7$	(L)	$3\sin x + 4\sin y + 7\sin z = 6$ (with X, Y, Z written above the sines)	(L)
$x + 3\sqrt{y} = 0$	(NL)		

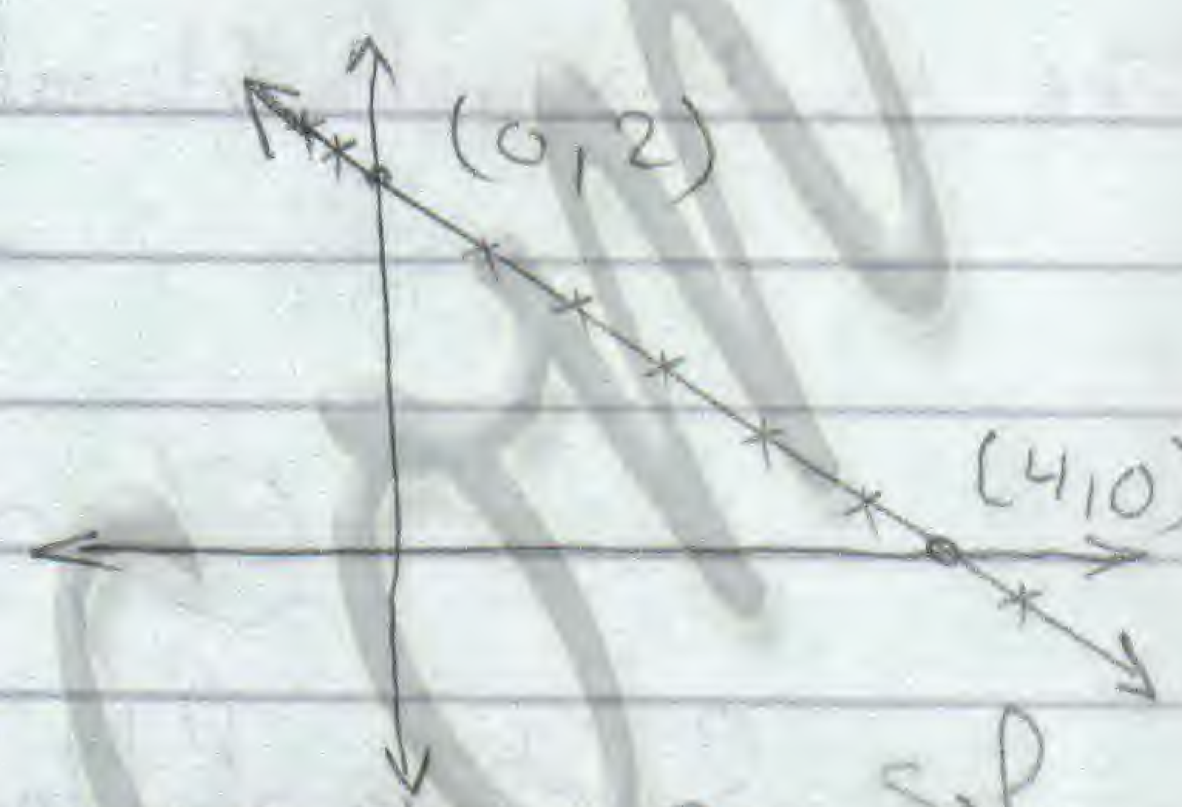
Soln. Of Linear Equations

Ex. 1 $x + 2y = 4$

Graphically

at $x = 0 \rightarrow y = 2$ $(0,2)$

at $y = 0 \rightarrow x = 4$ $(4,0)$



more than One Soln.

Using Linear Alg. ∴ let $\boxed{y = t}$ ∴ $\boxed{x = 4 - 2t}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} t + \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

Ex. 2 find the Solution. $x + 2y + 3z = 12$

∴ let $z = t$ $y = s$

$x = 12 - 2s - 3t$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} t + \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

∴ The Ans. has more than One Soln

Ex. 3

$x + y = 4$

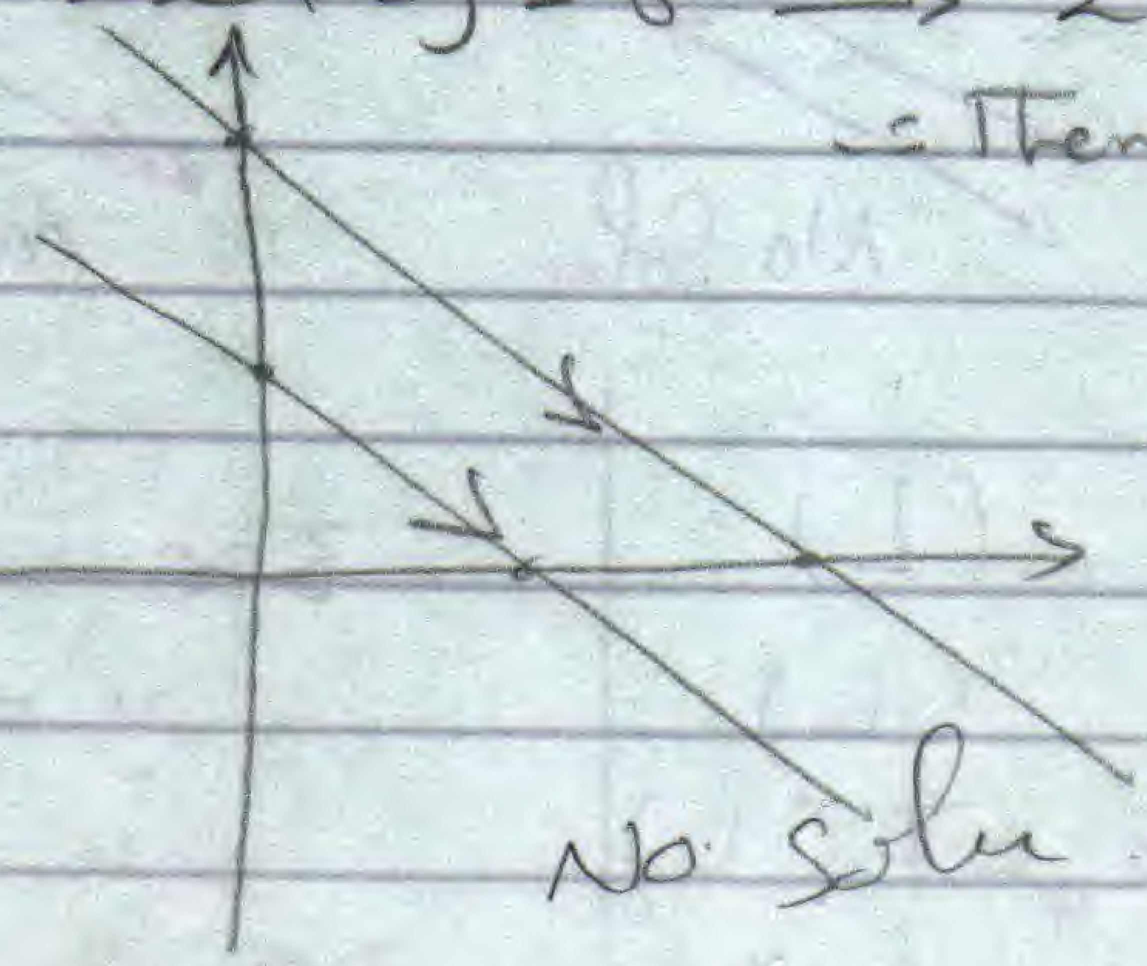
$2x + 2y = 6$

∴ Soln. by Calculus $x + y = 4 \rightarrow 1 + y' = 0 \rightarrow \boxed{y' = -1}$

$2x + 2y = 6 \rightarrow 2 + 2y' = 0 \rightarrow \boxed{y' = -1}$

∴ There is no Soln.

by Graph



$-2r_1+r_2 \to r_2$
$$\begin{array}{cc} x & y & \\ \left[\begin{array}{cc|c} 1 & 1 & 4 \\ 2 & 2 & 6 \end{array}\right] \end{array}$$

Augmentation Matrix

$$\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & 0 & -2 \end{array}\right]$$

$0 \neq -2$ ∴ There is no Soln.

Ex 4

$x + y = 4$
$x - y = 4$

by Calculus

$\rightarrow \quad y' = -1$

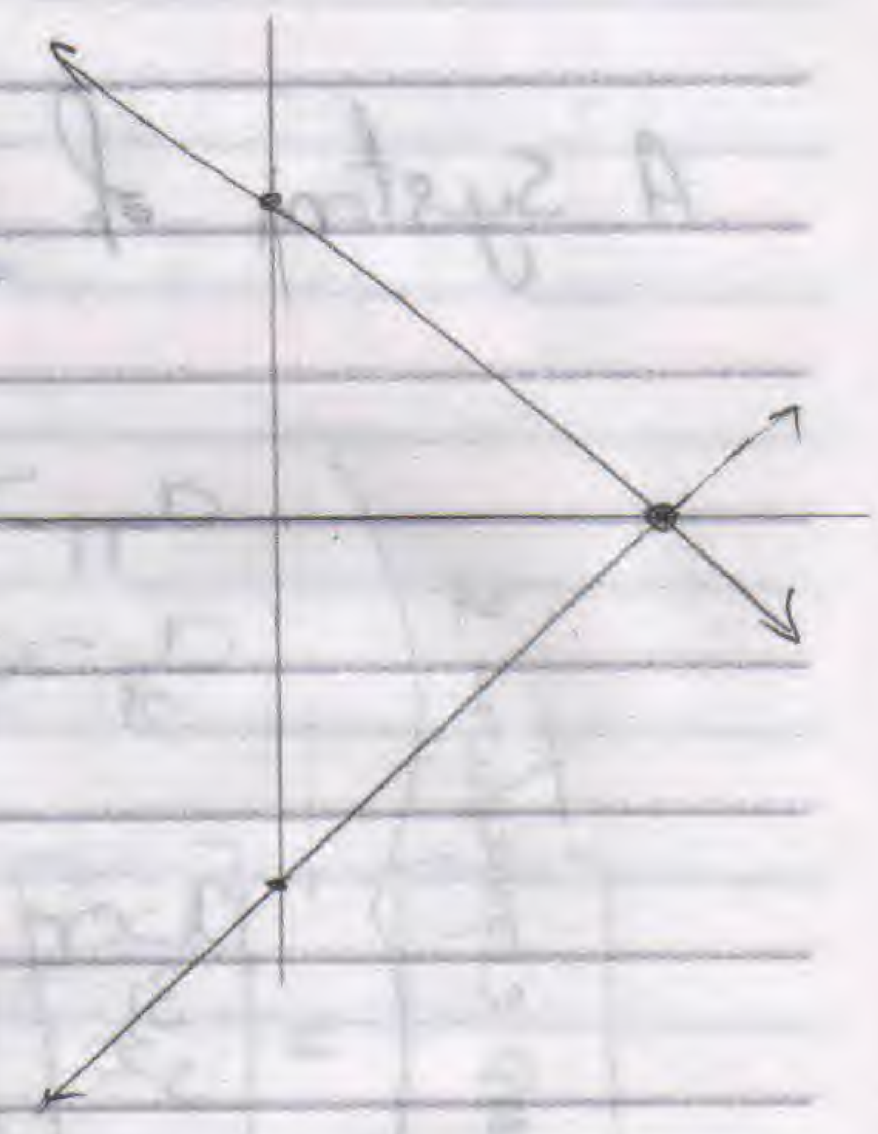
$\rightarrow \quad 1 - y' = 0 \Rightarrow y' = 1$

There is only one Soln.

by Graph,

1^{st} line $\Rightarrow$ $x = 0 \rightarrow y = 4$, $y = 0 \rightarrow x = 4$

2^{nd} line $x = 0 \rightarrow y = -4$, $y = 0 \rightarrow x = 4$



by Li. Alg.

$-r_1 + r_2 \to r_2$
$$\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 1 & -1 & 4 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & -2 & 0 \end{array}\right]$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$-2y = 0 \Rightarrow \boxed{y = 0}$

$x + y = 4 \rightarrow \boxed{x = 4}$ ∴ There is Only One Soln.

Ex.5 $x+y=4$

$2x+2y=8$

$$-2r_1+r_2\to r_2\quad\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 2 & 2 & 8 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & 0 & 0 \end{array}\right]$$

$x+y=4$

∴ let $\boxed{y=t}$ $\boxed{x=4-t}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} t + \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

∴ It has more one solu.

Augmented Matrices

A system of **m** linear equations in **n** unknowns

$$m \text{ equations}\left\{\begin{array}{l} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \text{——} + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \text{——} + a_{2n}x_n = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \text{——} + a_{3n}x_n = b_3 \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \text{——} + a_{mn}x_n = b_m \end{array}\right.$$

Can be written Only the Rectangular array of numbers:

$$\begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & \text{——} & x_n \end{array} \\ \left[\begin{array}{cccc|c} a_{11} & a_{12} & a_{13} & a_{1n} & b_1 \\ a_{21} & a_{22} & a_{23} & a_{2n} & b_2 \\ a_{31} & a_{32} & a_{33} & a_{3n} & b_3 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & a_{mn} & b_m \end{array}\right] \end{array}$$

we called that using Gauss eliminati

Ex: Find the augmented Matrix for the System of equations

$$x_1 + x_2 + 2x_3 = 9$$
$$2x_1 + 4x_2 - 3x_3 = 1$$
$$3x_1 + 6x_2 - 5x_3 = 0$$

$-2r_1 + r_2 \rightarrow r_2$
$-3r_1 + r_3 \rightarrow r_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{array}\right]$$

$-3r_2 + 2r_3 \rightarrow r_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 3 & -11 & -27 \end{array}\right]$$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 0 & -1 & -3 \end{array}\right]$$

$$-x_3 = -3 \Rightarrow \boxed{x_3 = 3}$$
$$2x_2 - 7x_3 = -17$$
$$2x_2 - 21 = -17 \Rightarrow \boxed{x_2 = 2}$$
$$x_1 + x_2 + 2x_3 = 9$$
$$x_1 + 2 + 2(3) = 9 \Rightarrow \boxed{x_1 = 1}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

** We Can Solve the System of Linear Equations by a methode called "Elementry row equations", "The Gauss Elimination"

** Three Operations correspond the following operations on the Rows of the augmented Matrix

1. Multiply a row through by a non Zero Constant
2. Interchange two rows
3. Add a multiple of One row to another row

$\underline{Ex._7}$ Using Element Row Operations (Gauss Elimination) to Solve The System equation.

$$x + y + 2z = 9$$
$$2x + 4y - 3z = 1$$
$$3x + 6y - 5z = 0$$

$-2r_1+r_2 \to r_2$
$-3r_1+r_3 \to r_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{array}\right] \xrightarrow{-3r_2+2r_3 \to r_3} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 3 & -11 & -27 \end{array}\right]$$

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 0 & -1 & -3 \end{array}\right] \end{array}$$

$\therefore\ -z = -3 \Rightarrow \boxed{z=3}$

$\therefore\ 2y - 7z = -17$

$2y = -17 + 7(3) \Rightarrow \boxed{y=2}$

$\therefore\ x + y + 2z = 9$

$x + 2 + 6 = 9 \Longrightarrow x = 9 - 8 \Rightarrow \boxed{x=1}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ $\therefore$ it has Just One Solu.

(It is Same as ex.6)

$\underline{Ex._8}$

$$3x_1 - 3x_2 = -2$$
$$2x_1 + x_2 = 1$$
$$3x_1 + 2x_2 = 1$$

$$\xrightarrow[-3r_2+2r_3 \to r_3]{-2r_1+3r_2 \to r_2} \left[\begin{array}{cc|c} 3 & -3 & -2 \\ 2 & 1 & 1 \\ 3 & 2 & 1 \end{array}\right] \Rightarrow -r_2+9r_3 \to r_3 \left[\begin{array}{cc|c} 3 & -3 & -2 \\ 0 & 9 & 7 \\ 0 & 1 & -1 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 3 & -3 & -2 \\ 0 & 9 & 7 \\ 0 & 0 & -16 \end{array}\right]$$

is $0 \neq -16$ ∴ There is no Soln.

Ex. 9

$$5x_1 - 2x_2 + 6x_3 = 0$$
$$-2x_1 + x_2 + 3x_3 = 1$$

$$\xrightarrow{2r_1+5r_2 \to r_2} \left[\begin{array}{ccc|c} 5 & -2 & 6 & 0 \\ -2 & 1 & 3 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 5 & -2 & 6 & 0 \\ 0 & 1 & 27 & 5 \end{array}\right]$$

let $\boxed{x_3 = t}$

∴ $x_2 + 27x_3 = 5 \Rightarrow \boxed{x_2 = 5 - 27t}$

$$5x_1 - 2x_2 + 6x_3 = 0$$
$$5x_1 - 2(5-27t) + 6t = 0$$
$$x_1 = \frac{1}{5}\left[10 - 54t - 6t\right]$$
$$\boxed{x_1 = 2 - 12t}$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -12 \\ -27 \\ 1 \end{bmatrix} t + \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$$

∴ it has more than One Soln.

$$\begin{bmatrix} u \\ v \\ w \\ x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ 3 \\ 1 \\ 1 \end{bmatrix} t + \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} M + \begin{bmatrix} -16 \\ 0 \\ 11 \\ 4 \\ 0 \end{bmatrix}$$

$\therefore$ It has more than one soln.